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1 July 1979 - 30 June 1980

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SOL R. BODNER

MML Report No. 70

MATERIAL MECHANICS LABORATORY
FACULTY OF MECHANICAL ENGINEERING
TECHNION—ISRAEL INSTITUTE OF TECHNOLOGY
HAIFA, ISRAEL



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Sol R. Bodner\* Principal Investigator

## Abstract

This report reviews the program of work performed under Contract No. F49620-79-C-0196, sponsored by the United States Air Force Office of Scientific Research through the European Office of Aerospace Research and Development (EOARD) during the period 1 July 1979 to 30 June 1980. A principal subject of the research program was the development of the anisotropic hardening theory of Stouffer and Bodner for elastic-viscoplastic materials into a practical computational tool. This has been achieved and demonstrated for problems of the dynamic impact of indenters into slabs of the reference material which experience induced plastic anisotropy. Another development has been the application of the elastic-viscoplastic constitutive equations with isotropic hardening to problems involving laminated materials. The equations have also been used as a basis to examine the response of metals over a wide range of temperatures and strain rates and as the material model in fracture mechanics problems.

Professor, Faculty of Mechanical Engineering Technion - Israel Institute of Technology

# <u>Objectives</u>

In the past ten years, considerable progress has been made by the Technion's Material Mechanics Laboratory to develop a new approach in modelling the elastic-viscoplastic behavior of materials for application in problems of stress analysis. The basic constitutive equations were formulated by Bodner and Partomin a series of papers and a number of studies were made to determine the applicability of those equations. Some of the main characteristics of the equations are that they do not require a yield criterion or loading/unloading conditions, inherently include strain-rate and time dependent effects, and are readily generalized to apply over a temperature range. For the most part, the basic equations are capable of adequately predicting material response for a wide range of loading and temperature circumstances when the conditions of isotropic hardening are essentially realized.

In order that the constitutive equations would apply for completely general loading paths, Stouffer and Bodner (1979) generalized the formulation to include anisotropic hardening induced by the plastic deformation. The resulting theory is somewhat complicated and one of the main objectives of the present program was to develop the Stouffer-Bodner theory into a practical working procedure for the solution of problems.

The class of problems chosen for this effort was the dynamic penetration of rigid indentors into slabs of the elastic-viscoplastic material. Indentation problems have received considerable attention because of their direct relation to applications and also due to

their special mathematical features. Although a considerable literature exists on indentation problems for elastic slabs, very little work has been done for problems involving ideally plastic materials and nothing at all for elastic-viscoplastic media. Attention was therefore directed toward solving the dynamic indentation of slabs of elastic-viscoplastic material that exhibit either isotropic or anisotropic hardening.

In addition to the effort to generalize the constitutive equations for arbitrary loading paths by considering anisotropic hardening, attention was directed toward including temperature dependence in the equations with isotropic hardening and to examine specific geometries of interest. The constitutive equations could be considered to be unified creep-plasticity equations in that both inelastic phenomena are represented by the same equations. A study was made on the response of copper and aluminum for a wide range of strain rates at some elevated temperatures. An earlier investigation used the basic equations for both plasticity and creep deformations of René 95 at 1200°F and showed their applicability over 10 decades of time.

A special geometrical form for applying the constitutive equations is that of a slab of periodically laminated materials which are elastic-viscoplastic. Consideration of the overall response of such a medium essentially involves the development of a mixture theory for a laminated composite. This was another objective of the program. A mixture theory for this case is considered to be relevant in analyzing the mechanical response of metal matrix composites.

Another application of the equations is the analysis of stress and strain fields around cracks in elastic-viscoplastic materials. A subject that initiated in the previous research program with AFOSR is the incorporation of the constitutive equations into finite element programs. An extension of that work is the use of a finite element program to determine the inelastic stress fields around cracks as a function of the rate of application of the external load. The overall aim of this study is to obtain the rate dependence of fracture toughness values. The approach employed is to calculate the stresses in the plastic zone near the crack tip and to use a local failure criterion to determine the conditions for the growth of the crack.

## Personnel

In addition to the Principal Investigator, Professor S.R. Bodner, the following persons were engaged on the research program:

Associate Professor Jacob Aboudi (Tel Aviv University)

Dr. Anthony Merzer (Technion)

Dr. Yako Benveniste (Tel Aviv University)

Mr. Zvi Zaphir (Graduate Student)

Prof. Bodner was a Visiting Professor at the University of Illinois, Urbana, IL from September 1979 until January 1980.

Prof. Aboudi has been a Visiting Scientist at Northwestern

University, Evanston, IL, for the academic year 1979-80.

## Publications of Research Program

1. "The Dynamic Indentation of an Elastic-Viscoplastic Work-Hardening Slab by a Rigid Punch," J. Aboudi (communicated by S.R. Bodner), International Journal of Engineering Science, vol. 18, 1980, pp 619-629.

Abstract - A unified theory for elastic-viscoplastic work-hardening materials, which requires neither a yield criterion nor loading or unloading conditions, is implemented to solve two-dimensional dynamic problems. Specifically, the theory is applied to the dynamic indentation by a rigid indenter of a slab made of an elastic-visoplastic material. The contact area between the indenter and the slab at any time is not known in advance but should be determined from the process of the solution. An iterative numerical procedure is proposed by which the complete solution is determined from the dynamic elastic-viscoplastic equations, the moving boundary conditions, the requirement that the contact normal stresses are compressive and that no interpenetration occurs outside the contact area. The method is applied to the indentation problem of a viscoplastic slab by a long rigid circular cylinder, and by a wedge-shaped die. Comparisons with the corresponding perfectly elastic problems are given.

2.\* "Dynamic Response of a Slab of Elastic-Viscoplastic Material that Exhibits Induced Plastic Anisotropy," J. Aboudi and S.R. Bodner, International Journal of Engineering Science (in press). (Presented at the Conference of the Society for Engineering Science, Northwestern University, Sept. 1979).

Abstract - Various two-dimensional problems of the dynamic loading of a slab are solved for a material characterization that is elasticviscoplastic and exhibits anisotropic work hardening. The governing constitutive equations are based on a unified formulation which requires neither a yield criterion nor loading or unloading conditions. They include multi-dimensional anisotropic effects induced by the plastic deformation history. The theory also considers plastic compressibility which depends on the extent of anisotropy. A numerical procedure for solving the equations is developed which incorporates the history dependent anisotropic hardening effects. Cases considered are the dynamic penetration of a slab by a rigid cylindrical indenter, and a distributed force rapidly applied over part of the slab surface. Both conditions of fixed and free rear surfaces of the slab are examined. A uniaxial problem is also considered in which different bases for the anisotropic hardening law are examined.

<sup>\*</sup> This paper is based on Scientific Report No. 1 (MML Report No. 67) issued August 1979

3. "Implementation of Elastic-Viscoplastic Constitutive Equations into "NONSAP" with Applications to Fracture Mechanics,"
Z. Zaphir and S.R. Bodner, Proceedings of the Conference on Nonlinear Finite Element Analysis and ADINA, K.J. Bathe (Ed), MIT Report 82448-9, August 1979.

Abstract - A set of elastic-viscoplastic constitutive equations formulated by Bodner and Partom has been incorporated into the "NONSAP" finite element program. The material characterization inherently includes the properties of strain rate sensitivity of plastic flow, work hardening, and reversed (cyclic) loading behavior. Features of these constitutive equations relevent to finite element programs are the applicability of a single set of equations for all loading/unloading conditions. As a consequence, the computational logic for non-monotonic and cyclic loading is considerably simpler than that for the classical elastic-plastic model.

The "NONSAP" finite element program with the adopted material characterization has been utilized to study the stress field surrounding a crack in an elastic-viscoplastic, work hardening medium. In particular, the DCB geometry was examined for conditions of varying external velocities and forces applied to the system. The complete stress field in the vicinity of the crack was calculated for different values of the force and velocity. A possible method for utilizing these results to obtain a rate dependent fracture initiation criterion is described.

4. "An Average Theory for the Dynamic Behavior of a Laminated Elastic-Viscoplastic Work-Hardening Medium," J. Aboudi and Y. Benveniste. (submitted to the Journal of the Mechanics and Physics of Solids).

Abstract - An average theory which models the dynamic behavior of a bilaminated medium under specific types of loading, made of elastic-viscoplastic work-hardening materials is developed. Each constituent is represented by a unified theory of elastic-viscoplasticity including work-hardening which requires neither a yield criterion nor loading or unloading conditions. The resulting effective theory appears in the form of a system of nonlinear differential equations for the average stresses, displacement and plastic work. The theory is applied to construct the effective stress-strain curves of the laminated medium in which the transition from the elastic to the plastic domain can be determined. The theory is also applied to obtain the dynamic response of a laminated slab subjected to velocity or stress input.

5. "An Average Theory for the Dynamic Behavior of a Laminated Elastic-Viscoplastic Medium Under General Loading," J. Aboudi and Y. Benveniste, Internation Journal of Solids and Structures (in press).

Abstract - An average theory which models the dynamic behaviour of a bi-laminated composite medium made of elastic-viscoplastic work hardening constituents is presented. The resulting effective theory is represented by a system of nonlinear differential equations for the average stresses, displacements, and the plastic work. The theory can be applied to three-dimensional problems under general types of loading. The theory is applied for the special cases of waves propagating normal to the layering and for waves propagating in a thin composite rod.

# Publications in Preparation

- 1. "Transient and Steady Creep Behavior Predicted by Unified Constitutive Equations," A. Merzer.
- 2. "Plastic Wave Propagation Based on an Anisotropic Work Hardening Theory," S.R. Bodner.

# Publication during current year of results obtained in previous research program with AFOSR (Grant AFOSR-75-2607)

- 1. "Projectile Perforation of Multi-Layered Beams," I. Marom and S.R. Bodner, International Journal of Mechanical Sciences, vol. 21, 1979, pp 489-504.
- 2. "Analytical Formulation of a Rate and Temperature Dependent Stress-Strain Relation," A. Merzer and S.R. Bodner, Journal of Engineering Materials and Technology, Trans. ASME, vol. 101, 1979, pp 254-257.
- 3. "Analytical and Computational Representation of High Rate of Straining Behaviour," A. Merzer and S.R. Bodner, Proc. Conference on The Mechanical Properties of Materials at High Rates of Strain, Conference Series No. 47, Institute of Physics, London, 1979, pp 142-151.
- 4. "Uniaxial Cyclic Loading of Elastic-Viscoplastic Materials," S.R. Bodner, I. Partom and Y. Partom, Journal of Applied Mechanics, Trans. ASME, vol. 46, 1979, pp 805-810. (Presented at the Annual Meeting of the ASME, New York, Dec. 1979).

# <u>Accomplishments</u>

As a first step in analyzing problems of the dynamic indentation of slabs of elastic-viscoplastic materials, the case of isotropic hardening was treated by Aboudi (publication no.1). The loading condition consisted of prescribing the velocity at which a rigid punch of a particular geometry would indent the slab. Inertia terms were included in the equations of motion so that both elastic and plastic wave effects were obtained. Calculations were performed using a finite difference scheme at progressive time increments for both elastic and elastic-viscoplastic media for comparison.

The results showed, as expected, appreciable differences in the stress and deformation response for the two cases. For the viscoplastic slab, the results depended strongly on the imposed velocity of the indenter and approached the elastic values with increasing velocities. Unloading, i.e., reversal of the indenter velocity, was also studied, and the overall results indicated very different material response characteristics for the elastic and viscoplastic media.

The major effort was given to analyzing dynamic indentation problems in which the slab material was characterized by the Stouffer-Bodner anisotropic hardening theory (Aboudi and Bodner, publication no. 2). In this case, the material develops anisotropic properties with regard to resistance to plastic flow as a consequence of the plastic deformation. The theory appears to be a much more

realistic material model than the isotropic hardening theory when general loading paths are considered. An interesting consequence of the theory is that plastic volume changes are non-zero and are obtained as part of the solution. In addition to solving a number of problems of dynamic indentation and force application on slabs, some exercises were performed on uniaxial wave propagation in slabs. These uniaxial wave problems were also used to examine different bases for the hardening law, i.e., whether the current direction of plastic strain-rate or stress should be taken as the reference for relative hardening along different axes.

An important achievement of this exercise was that the anisotropic hardening theory can be meaningfully used to calculate stress and strain histories for various multi-dimensional dynamic problems. When the current stress is used as the reference in the anisotropic hardening law, the effort involved in the calculations is not much greater than that for isotropic hardening which, in turn, is similar to that for the classical ideally plastic solid. The results showed that induced plastic anisotropy can have a strong influence on the stress levels. In the numerical examples, differences with the isotropic hardening law generally occurred with the onset of reflected loading waves.

A separate program was undertaken by Merzer and Bodner to examine the applicability of the constitutive equations at high temperatures under conditions of plastic straining, creep and stress relaxation. The reference materials were taken to be copper and

aluminum for which a fair amount of test data is available, For aluminum, an earlier experimental investigation at the Technion provided information on creep testing under changing loads, i.e., increments and decrements in load superimposed on a steady value. Results to date indicate that the constitutive equations with isotropic hardening and inclusion of a hardening recovery term can provide good representation and predictability for these conditions. A report on this work is in preparation.

Another application of the constitutive equations with isotropic hardening was in calculating the inelastic stress fields around cracks as a function of loading rate. This was achieved by adapting the material model to the "NONSAP" finite element program which was used for the caluclations. A report on this study was given by Zaphir and Bodner (publication no. 3). Subsequent work on this problem has been directed toward examining unloading behavior and repeated loading which would be of considerable interest to the fatigue crack growth problem.

Two papers were prepared by Aboudi and Benviniste (publications nos. 4 and 5) on the development of a mixture theory for laminated slabs consisting of two metals in periodic layers, each of which is characterized by the elastic-viscoplastic constitutive equations. An overall effective theory for the composite medium was developed which gives equations for the average stresses and displacements. Effective stress-strain curves for the laminated slab can be calculated from those equations. They have also been solved for a number of two and three dimensional dynamic loading conditions.

## LIST OF PAPERS ON THE APPLICATION OF THE UNIFIED THEORY OF VISCOPLASTICITY

### Application for Dynamic Indentation and Wave Propagation:

- (1) J. Aboudi, "The Dynamic Indentation of an Elastic-Viscoplastic Work-Hardening Slab by a Rigid Punch". Int. J. Eng. Sci., 18, 619-629, 1980.
- (2) J. Aboudi and S.R. Bodner, "Dynamic Response of a Slab of Elastic-Viscoplastic Material that Exhibits Induced Plastic Anisotropy", Int. J. Eng. Sci., <u>18</u>, 801, 1980.

### Applications on Composite Materials

- (3) J. Aboudi and Y. Benveniste, "An Average Theory for the Dynamic Behavior of a Laminated Elastic-Viscoplastic Medium Under General Loading", Int. J. Solids Structures (in press).
- (4) J. Aboudi and Y. Benvenisti, "An Average Theory for the Dynamic Behavior of a Laminated Elastic-Viscoplastic Work-Hardening Medium", (submitted for publication).
- (5) J. Aboudi, "Effective Stiffness Theory for a Laminated Elastic-Viscoplastic Work-Hardening Composite", Int. J. Solids Structures (in press).
- (6) J. Aboudi, "Generalized Effective Stiffness Theory for Nonelastic Laminated Composites", (submitted for publication).

### Applications for Cracks

- (7) J. Aboudi and J.D. Achenbach, "Rapid Mode III Crack Propagation in a Strip of Viscoplastic Work-Hardening Material". Int. J. Solids Structures (in press).
- (8) J. Aboudi and J.D. Achenbach, "Numerical Analysis of Fast Mode-I Fracture of a Strip of Viscoplastic Work-Hardening Material", (submitted for publication).
- (9) J. Aboudi and J.D. Achenbach, "Fast Fracture of a Strip of Viscoplastic Work-Hardening Material", Proc. of Int. Conf. on Analytical and Experimental Fracture #echanics (in press).
- (10) J. Aboudi and J.D. Achenbach, "Arrest of Mode-III Fast Fracture by a Transition from Elastic to Viscoplastic Material Properties" (submitted for publication).